

Constrained Flavor Breaking

Thomas Appelquist,¹ Yang Bai,² and Maurizio Piai³

¹*Department of Physics, Sloane Laboratory, Yale University, New Haven, Connecticut 06520, USA*

²*Department of Physics, University of Wisconsin, Madison, WI 53706, USA*

³*Swansea University, College of Science, Singleton Park, Swansea, Wales, UK*

We explore predictive flavor models based on subgroups of the standard-model $SU(3)^5$ flavor symmetry. Restricting to products of $SU(3)$, we find that a global $SU(3)^3$ flavor symmetry, broken only by two Yukawa spurions, leads to a relation among down-type quark, up-type quark and charged-lepton Yukawa matrices: $Y_d \propto Y_u Y_e^\dagger$. As a result, the charged-lepton mass ratios are expressed in terms of quark mass ratios and mixing angles. Large leptonic mixing angles appear to be natural and lead to contributions to flavor-changing neutral currents in the charged-lepton sector, which can be tested in future precision experiments.

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I. INTRODUCTION

The standard model (SM) of particle physics provides a very successful description of physics up to the TeV scale. In particular, the simplest realization of electroweak symmetry breaking, in which the Higgs mechanism is implemented via one or more scalar fields, is in excellent agreement with the properties of the recently discovered Higgs particle [1, 2]. The masses of all the elementary fermions arise after electroweak symmetry breaking due to the presence of Yukawa interactions. The fundamental origin of the Yukawa couplings remains unknown. All attempts at deriving them from a more fundamental theory of flavor involve the observation that in the absence of the Yukawa couplings, the SM Lagrangian possesses an enlarged $G = SU(3)^5$ global symmetry.¹ This symmetry is ultimately responsible for the naturalness of the Yukawa couplings and fermion masses, and it is the origin of the suppression of flavor-changing neutral current (FCNC) transitions.

There exists a vast literature on models of flavor generation, differing from one another by what subgroup of G plays a role in the underlying theory of flavor [3, 4], and what mechanism(s) yield its breaking. At one extreme sit models with minimal flavor violation (MFV) [5], in which one assumes that in the ultraviolet (UV) completion of the SM, the $SU(3)^5$ symmetry is violated only by the SM Yukawa couplings. Hence all FCNC processes are suppressed by combinations of the Yukawa couplings (see for example [6]). An advantage of MFV is that it yields a restrictive structure for the possible operators controlling FCNC transitions. As a result the mass scale associated with these processes can be relatively low, in the TeV range.

The MFV framework does not, however, yield any new testable predictions for the mass and mixing matrices appearing in the SM. Hence, it does not reveal anything new about the origin of flavor. The masses, mixing angles, and CP-violating phases are treated as independent free parameters.

In this paper, we investigate whether it is possible to gain some insight into the origin of flavor in an MFV-like framework by constraining more tightly the Yukawa-coupling matrices that break the flavor symmetry. Our constrained-flavor-breaking (CFB) approach is presented as an effective field theory (EFT) valid only up to the flavor-breaking scale F . We obtain two relations for charged-lepton mass ratios in terms of the quark masses and mixing angles. Using the smallness of both CKM mixing angles and quark mass ratios, the symmetry structure enforces large off-diagonal entries in the lepton mass matrix, which suggest the natural appearance of large leptonic mixing angles.

We describe our framework in section II, and our specific model in section III, focusing on the terms in the EFT Lagrangian responsible for breaking the underlying flavor symmetry and generating the quark and charged lepton masses. We then explore the resultant relations among the Yukawa matrices. We discuss higher-dimension operators responsible for FCNC transitions in section IV, coming to the conclusion that leptonic FCNC processes constrain the scale of the higher-dimension operators to be above 10^4 TeV. We summarize our results and discuss open questions in section V. In appendix A, we present some notation and list the measured values of quark and lepton masses and CKM mixing angles.

II. FRAMEWORK

We consider only the quark and charged-lepton sectors, leaving neutrino mass and mixing for future study. The maximal global non-Abelian flavor symmetry in the absence of mass is then $SU(3)^5$. We break the flavor symmetries with dimensionless spurion fields (Yukawa matrices) that are bi-fundamentals under two of the

¹ The symmetry can be extended to $G = SU(3)^6$ if one assumes the existence of 3 right-handed neutrinos. We do not do so here. There are also a set of $U(1)$ symmetries to consider. Among these, we make direct use of only lepton number and baryon number conservation.

$SU(3)$'s. A minimum of three spurions would be required to break all five of the $SU(3)$'s: two (Y_d and Y_u) to break $SU(3)_{Q_L} \times SU(3)_{d_R} \times SU(3)_{u_R}$ and one (Y_e) to break $SU(3)_{L_L} \times SU(3)_{e_R}$. As a result, there are no predictions for masses and mixing angles with $SU(3)^5$.

A predictive framework can allow at most two bi-fundamental spurions. This suffices to give mass to all the fermions if there are at most four global $SU(3)$'s, with at least two of the five fermion fields transforming under the same $SU(3)$. With four $SU(3)$'s one spurion can break $SU(3)_{L_L} \times SU(3)_{e_R}$, while the other breaks $SU(3)_{Q_L} \times SU(3)_{u_R, d_R}$. This model, though, predicts $Y_d \propto Y_u$, giving incorrect quark mass ratios and vanishing CKM mixing angles. Other assignments for $SU(3)^4$ can also be seen to yield unrealistic results.

We therefore turn to a global $SU(3)^3$ symmetry, with two spurions for the flavor breaking. To have realistic quark masses and mixing angles, we assign Q_L , d_R and u_R each to a different one of the $SU(3)$'s. There are then six options to assign L_L and e_R to two different $SU(3)$'s.

For the four cases with either L_L or e_R transforming under $SU(3)_{Q_L}$, one has the unrealistic prediction that the charged-lepton mass ratios are proportional to either the up-type or down-up quark mass ratios. The remaining two cases correspond to $SU(3)_{Q_L} \times SU(3)_{d_R, L_L} \times SU(3)_{u_R, e_R}$ or $SU(3)_{Q_L} \times SU(3)_{d_R, e_R} \times SU(3)_{u_R, L_L}$.

In this paper, disregarding the neutrinos and focusing on only the charged-lepton mass ratios in the leptonic sector, these two cases are equivalent because the charged-lepton mass matrices are the Hermitian transpose of each other. We adopt the assignment $SU(3)_{Q_L} \times SU(3)_{d_R, L_L} \times SU(3)_{u_R, e_R}$, as shown in the moose diagram of Fig. 1 and in Table I, where the three $SU(3)$'s are labeled 1, 2, and 3.

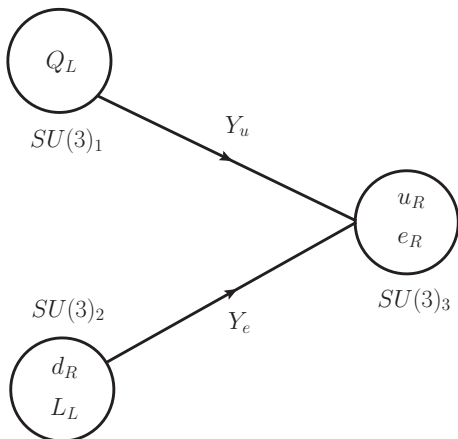


FIG. 1: A moose diagram for our model.

We next choose the $SU(3)$ assignments for the two bi-fundamental spurions, which we take to be SM singlets. The possibility we focus on, shown in Table I, is to employ a Y_u transforming under $SU(3)_1 \times SU(3)_3$ and a Y_e transforming under $SU(3)_2 \times SU(3)_3$. This provides

independent mass matrices for the up-type quarks and charged leptons. The down-type quark mass matrix will require an EFT term with a product such as $Y_u Y_e^\dagger$, leading to a relation among the lepton and up-type quark mass ratios and CKM mixing angles.

There are two other choices for the two bi-fundamental spurions. One, with Y_u transforming under $SU(3)_1 \times SU(3)_3$ and Y_d transforming under $SU(3)_1 \times SU(3)_2$, would lead to the charged lepton mass matrix arising from a product such as $Y_d^\dagger Y_u$. In light of the small mixing angles in the CKM matrix, this gives the unrealistic result of the charged-lepton masses being more hierarchical than the up-type quark masses. A third choice, with Y_d transforming under $SU(3)_1 \times SU(3)_2$ and Y_e transforming under $SU(3)_2 \times SU(3)_3$, also has a difficulty of obtaining a large enough top Yukawa coupling, at least in models with a single Higgs doublet.²

A restriction to fewer than three global flavor $SU(3)$'s leads to an over constrained flavor structure. In Ref. [7], for example, two $SU(3)$'s are employed for the quarks and charged leptons, allowing for compatibility with $SU(5)$ grand unification: one for the $\bar{5}$ (L_L , d_R^c) and one for the 10 (Q_L , u_R^c , e_R^c). This leads to quark and lepton mass relations far from the experimental values, addressed in Ref. [7] by the introduction of additional spurions. Our approach, with three flavor $SU(3)$'s provides a more realistic description of fermion masses and mixing angles, but it isn't yet clear to us how to embed it into a larger framework of gauge unification.

The origin of the matrices Y_u and Y_e is not specified here. One could imagine, for example, that Y_u emerges from an underlying dimension-five operator of the form $\eta_u \tilde{H} \bar{Q}_L \Phi_u u_R / F$, where \tilde{H} is the conjugate of the SM Higgs doublet, η_u is a dimensionless coupling and Φ_u is a dimension-one dynamical field with a vacuum expectation value (VEV) of order the flavor breaking scale F . A similar term $\eta_e H \bar{L}_L \Phi_e e_R / F$ could lead to the charged-lepton mass matrix. The down-quark mass matrix could emerge from an even higher dimension operator, the simplest example being $\eta_d H \bar{Q}_L \Phi_u \Phi_e^\dagger d_R / F^2$.

III. THE MODEL AND FERMION MASS

The ingredients of our model with their assignments under the global flavor $SU(3)^3$ are shown in Table I. The list includes the quark and charged-lepton fields and the two dimensionless, bi-fundamental spurion fields Y_u and Y_e . The SM gauge fields and symmetries are not explicitly shown. The focus of this paper is on flavor breaking,

² All these relations are for (d_R, L_L) and (u_R, e_R) having the same representation. For the model with (d_R, L_L) and (u_R, e_R) having the complex-conjugate representations, one needs to replace Y_e by Y_e^* . Since this changes only the phases in the lepton sector, predictions of charged-lepton mass ratios are unaffected.

	$SU(3)_1$	$SU(3)_2$	$SU(3)_3$
H	1	1	1
Q_L	3	1	1
d_R	1	3	1
u_R	1	1	3
L_L	1	3	1
e_R	1	1	3
Y_u	3	1	$\bar{3}$
Y_e	1	3	$\bar{3}$

TABLE I: The global flavor symmetries and field content of the model.

not electroweak breaking, but for simplicity, for most of our discussion, we take electroweak breaking to be described by a single Higgs doublet $H \sim (1, 2)_{1/2}$ under the SM $SU(3)_{\text{QCD}} \times SU(2)_W \times U(1)_Y$ gauge symmetry. In this case, H develops the VEV

$$\langle H \rangle = \frac{v_W}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (1)$$

where $v_W \simeq 246$ GeV. We could equally well employ two Higgs doublets which, for large $\tan \beta$ (the ratio between the two vacuum expectation values), leads to a bottom-quark coupling comparable to the top-quark coupling. In either approach, the important question of the stability of the electroweak scale arises. We will not discuss it further in this paper as our focus is on the physics of flavor-symmetry breaking.

The quark and charged-lepton mass terms of the EFT are given by

$$- H \bar{Q}_L Y_d d_R - \tilde{H} \bar{Q}_L Y_u u_R - H \bar{L}_L Y_e e_R + \dots, \quad (2)$$

where the dots represent higher-dimensional operators, and where the matrix Y_d must be constructed from our two spurion fields Y_u and Y_e . The simplest structure based on the flavor symmetry is

$$Y_d = \eta Y_u Y_e^\dagger, \quad (3)$$

where η is a parameter to be chosen so that the largest entry in Y_d is $O(10^{-2})$ in order to fit the b -quark mass. This relation may appear counter-intuitive given that the mass hierarchy among the up-type quarks is much larger than the down-type quarks

An immediate question is whether this relation is stable in the face of quantum corrections. One-loop and higher corrections in the SM, for example from emission and re-absorption of the Higgs boson, will correct it, generating additive down-quark mass terms with matrix structures containing additional factors such as $Y_u Y_u^\dagger / (4\pi)^2$. These terms violate the matrix relation Eq. (3) and they typically enter with logarithmic UV cutoff dependence. But since no element of Y_u is larger than unity, these are small corrections for any reasonable cutoff. We return to this discussion in Section IV,

after introducing the higher-dimension operators of our EFT, which correct the dimension-4 terms of the SM Lagrangian.

After electroweak symmetry breaking, the mass matrices of the quarks and charged leptons are given by

$$M^{(d,u,e)} = \frac{1}{\sqrt{2}} Y_{d,u,e} v_W. \quad (4)$$

We can make $SU(3)_1$ and $SU(3)_2$ transformations to the basis in which Y_d is diagonal. Making further use of $SU(3)_3$ and using the convention in Eq. (A3), there is a basis in which we can write

$$Y_d = \text{diag}(Y_d), \quad Y_u = V_{\text{CKM}}^\dagger \text{diag}(Y_u). \quad (5)$$

Using Eq. (3) we then have

$$Y_e = \frac{1}{\eta} \text{diag}(Y_d) V_{\text{CKM}}^\dagger [\text{diag}(Y_u)]^{-1}. \quad (6)$$

Using the central values of the CKM angles, phase and quark mass parameters listed in Appendix A (and defined at the common scale M_Z), we find that in this special basis

$$|Y_e| \simeq \frac{1}{\eta} \begin{pmatrix} 1.8 & 0.0011 & 1.4 \times 10^{-7} \\ 8.2 & 0.093 & 0.000014 \\ 6.1 & 0.19 & 0.017 \end{pmatrix}. \quad (7)$$

The 21 entry of $|Y_e|$, given by $(1/\eta)|V_{\text{CKM}}^{12}|m_s/m_u$, is the largest. We thus infer that

$$m_\tau \simeq |Y_e|_{21} \frac{v_W}{\sqrt{2}} \simeq \frac{1}{\eta} \times (1.4 \times 10^3 \text{ GeV}), \quad (8)$$

requiring $\eta \simeq 10^3$ to fit the experimental τ mass.³

Despite the smallness of the CKM mixing angles, the Y_e matrix has some very large off-diagonal entries due to the large ratios m_s/m_u and m_b/m_u . The presence of $[\text{diag}(Y_u)]^{-1}$ in Eq. (6) magnifies the CKM angles. This

³ The large value of η must emerge from a UV completion of this model. Suppose, within the framework mentioned at the end of Section II, that $Y_e = \eta_e \langle \Phi_e \rangle / F$, with the largest entry of $\langle \Phi_e \rangle$ of $O(F)$ and $\eta_e \simeq 10^{-2}$. The matrix Y_d would similarly come from the operator $\eta_d H \bar{Q}_L \Phi_u \Phi_e^\dagger d_R / F^2$. Then, with the assumption of vacuum saturation, $Y_d = \eta_d \langle \Phi_u \rangle \langle \Phi_e^\dagger \rangle$. Here, the largest entry of $\langle \Phi_u \rangle$ is of $O(F)$, but the largest entry of the matrix product $\langle \Phi_u \rangle \langle \Phi_e^\dagger \rangle$ is of $O(10^{-3} F^2)$. Then $\eta_d \simeq \eta \eta_e \simeq O(10)$. It is also the case that corrections to Eq. (3) will emerge from whatever theory UV completes our EFT. Suppose, for example, that the Φ_u and Φ_e fields represent dynamical degrees of freedom within the UV completion. A naive estimate would lead to a correction of the form $Y_d \propto [Y_u + \epsilon_n (Y_u Y_u^\dagger)^n Y_u] Y_e^\dagger$, with $\epsilon_n = O\left\{\left[\frac{\eta_d M}{\eta_u (4\pi)^2 F}\right]^{2n}\right\}$, where M is the natural mass scale (cutoff) of the UV completion. This would substantially modify only the 33 element of Y_u entering Eq. (3) because of the hierarchical structure of Y_u . As long as $M < 4\pi F$, even the 33 element will receive only a small correction.

structure provides an unusual source for large mixing angles in a flavor model, and suggests that the large measured values of the PMNS angles are natural. We haven't discussed the neutrino mass matrix here, yet it would require a fine-tuned relation between Y_e and the neutrino mass matrix to avoid the presence of large mixing angles.

Most importantly, Eq. (6) leads to relations for the ratios of the charged-lepton masses. Diagonalizing the matrix Y_e and again using the central values of the CKM angles and quark masses, we obtain

$$\frac{m_e}{m_\tau}(\text{th}) \simeq 2.1 \times 10^{-4}, \quad (9)$$

$$\frac{m_\mu}{m_\tau}(\text{th}) \simeq 0.013, \quad (10)$$

to be compared with the experimental values

$$\frac{m_e}{m_\tau}(\text{exp}) \simeq 2.8 \times 10^{-4}, \quad (11)$$

$$\frac{m_\mu}{m_\tau}(\text{exp}) \simeq 0.059, \quad (12)$$

computed using the renormalized masses defined at the common scale M_Z (see Appendix A).

The experimental values are quite precise, but the theoretical relations are affected by the comparatively large errors in the measured values of the quark masses, particularly the light ones, the role of which is quite important. The central-value relation for m_e/m_τ is well within these uncertainties, while the central-value prediction of the muon mass appears to be within a factor of three of the experimental value. To be more precise, we have numerically varied the masses of the quarks and the CKM angles within the allowed experimental ranges, and compared the theoretical relations for the charged lepton mass ratios with the experimental data. This is illustrated in Fig. 2.

To provide some insight into our theoretical relations Eqs. (9,10), in particular their dependence on the quark mass ratios, we use the Wolfenstein parametrization of the CKM matrix Eq. (A5). Expressing the parameters ρ and η as $O(1)$ numbers times $\lambda \approx 0.23$ (approximately the Cabibbo angle), and taking the quark masses to be within roughly a factor of 2 of their experimental values, we find the approximate formulas

$$\frac{m_e}{m_\tau} \simeq \frac{m_d}{m_s} \frac{m_u}{m_t} \lambda^{-4}, \quad (13)$$

$$\frac{m_\mu}{m_\tau} \simeq \frac{m_b}{m_s} \frac{m_u}{m_c} \lambda. \quad (14)$$

We can see that to bring m_μ/m_τ closer to the experimental value would require increasing m_u or decreasing m_c while keeping the product $m_d m_u$ intact.

We are encouraged by these results. Large leptonic mixing angles appear to be natural. The charged-lepton mass ratios emerge from Eq. (6), in which CKM mixing plays an intricate and prominent role. In the absence of this mixing, the ratio m_μ/m_τ would be in good agreement with experiment, but m_e/m_τ would be predicted to be too large by more than an order of magnitude.

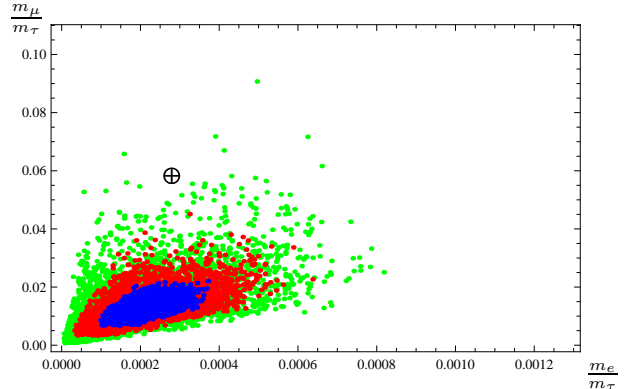


FIG. 2: Charged-lepton mass ratios in our model compared to the experimental data (black). The points are obtained by randomly choosing 4000 possible values of the masses of the quarks and CKM angles within the 3σ (green), 2σ (red) and 1σ (blue) ranges. Note that this scatter plot should not be interpreted as the result of a global fit.

IV. HIGHER DIMENSION SM OPERATORS AND FLAVOR-CHANGING NEUTRAL CURRENTS.

In the standard model, FCNC transitions are forbidden at the tree level due to the GIM mechanism, and arise only at the one-loop level. Due to the one-loop, weak-coupling suppression, the smallness of the CKM off-diagonal terms, and the quark mass hierarchies, all such processes are strongly suppressed. The underlying reason for the suppression is $SU(3)^5$ flavor symmetry, the breaking of which requires diagrams with the insertion of both the up-type and down-type Yukawa couplings. While this is an automatic consequence of the SM assignments of the fermions, the GIM mechanism is not a generic feature of extensions of the standard model.

This potential difficulty has been addressed by the MFV framework [6] in which *all* breaking of the $SU(3)^5$ flavor symmetry is taken to arise from the SM Yukawa couplings. The higher-dimension operators incorporating new physics beyond the SM and breaking the $SU(3)^5$ flavor symmetry, are suppressed by these factors.

In our model, with only an $SU(3)^3$ global flavor symmetry and two bi-fundamental spurions to implement the breaking, we have the special relation $Y_d \propto Y_u Y_e^\dagger$ among the Yukawa matrices for the quarks and charged leptons. In this section, we examine the resultant EFT constructed from SM fields, focusing on the higher-dimension operators that induce flavor-changing neutral currents. As in MFV, we build these operators from only the Yukawa matrices.

We construct them from dimension-three building blocks bilinear in the fermion fields, containing enough insertions of the Yukawa matrices Y_e and Y_u to respect

MFVu	MFVd	CFB
$O_{1u} = \bar{Q}_L Y_u Y_u^\dagger Q_L$	$O_{1d} = \bar{Q}_L Y_d Y_d^\dagger Q_L$	$O_{1e} = \bar{Q}_L Y_d Y_e Y_e^\dagger Y_d^\dagger Q_L$
$O_{2u} = \bar{Q}_L Y_u Y_u^\dagger Y_u u_R$	$O_{2d} = \bar{Q}_L Y_d Y_d^\dagger Y_u u_R$	$O_{2e} = \bar{Q}_L Y_d Y_e u_R$
$O_{3u} = \bar{Q}_L Y_u Y_u^\dagger Y_d d_R$	$O_{3d} = \bar{Q}_L Y_d Y_d^\dagger Y_d d_R$	$O_{3e} = \bar{Q}_L Y_d Y_e Y_e^\dagger d_R$
$O_{4u} = \bar{u}_R Y_u^\dagger Y_u u_R$	$O_{4d} = \bar{u}_R Y_u^\dagger Y_d Y_d^\dagger Y_u u_R$	$O_{4e} = \bar{u}_R Y_e^\dagger Y_e u_R$
	$O_{5d} = \bar{d}_R Y_d^\dagger Y_d d_R$	$O_{5e} = \bar{d}_R Y_e Y_e^\dagger d_R$
	$O_{6d} = \bar{u}_R Y_u^\dagger Y_d d_R$	$O_{6e} = \bar{u}_R Y_e^\dagger d_R$
	$O_{1e} = \bar{L}_L Y_e Y_e^\dagger L_L$	$O_{1\ell} = \bar{L}_L Y_d^\dagger Y_d L_L$
	$O_{2e} = \bar{L}_L Y_e Y_e^\dagger Y_e e_R$	$O_{2\ell} = \bar{L}_L Y_d^\dagger Y_u e_R$
	$O_{3e} = \bar{e}_R Y_e^\dagger Y_e e_R$	$O_{3\ell} = \bar{e}_R Y_u^\dagger Y_u e_R$

TABLE II: The building blocks (bilinear in SM quarks and charged leptons) contributing to higher order operators.

the flavor symmetry. For simplicity, we assume here that each building block by itself respects baryon and lepton number symmetry. One could relax this assumption, and include higher order operators that contain lepto-quark bilinears. As we will see, the bounds we find are already quite restrictive, sufficient to reach our main conclusions. The product $Y_u Y_e^\dagger$, when encountered, is replaced by Y_d as dictated by Eq. (3) to have a more conservative FCNC constraint. Each fundamental bilinear is then taken to enter with no further suppression by small parameters.

We ignore restrictions coming from the $SU(2)_L \times U(1)_Y$ gauge symmetry because we are now implementing the EFT below the EW breaking scale (the Kaon or B-meson scales, for example). We express these building blocks, without contracting color or spinor indexes. Thus several new higher-order operators having the same flavor structure, but different QCD or Lorentz structure, are treated on the same footing. We do not list here bilinears such as $\bar{Q}_L Q_L$ or $\bar{Q}_L Y_d d_R$ since it is always possible, at the low energies considered here, to write the theory in a basis in which the kinetic and mass terms are diagonal in flavor space.

The list of possible bilinears includes those that appear in MFV, such as quark bilinears constructed using only Y_u and Y_d . It also includes bilinears that do not appear in MFV: quark bilinears involving factors of Y_e and lepton bilinears involving Y_u and Y_d . In Table II, we list these bilinears, using only a minimal number of Yukawa matrices of each type. Columns 1 and 2 contain quark bilinears appearing in MFV. Those in column 2, containing more factors of Y_d (or Y_e) in each bilinear, are suppressed relative to those in column 1 in our single-Higgs-doublet model. With two Higgs doublets and large $\tan\beta$, some of them can be enhanced to the level of the column-1 bilinears. In column 3, we list bilinears that do not appear in MFV.

Not all the building blocks induce FCNC processes. In MFV with a single Higgs doublet, only bilinears quadratic in Y_u are important for FCNC processes (bilinears O_{1u} and O_{3u}). It turns out that O_{1u} for example contributes to all the main FCNC transitions mediated by operators of dimension 6 —such as Δm_K , ϵ_K , ΔM_{B_s} , $A_{CP}(B_d \rightarrow J/\psi K_S)$ and ΔM_{B_d} — while O_{3u} is important in processes dominated by dimension-5 operators (such as $b \rightarrow s \gamma$ and $B_s \rightarrow \mu^+ \mu^-$). The bilinears O_{2u}

and O_{4u} do not yield FCNC transitions.

All of the bilinears of columns 1 and 2 have been analyzed previously, coming to the conclusion when they are employed to construct dimension-5 and dimension-6 FCNC operators, the scale can be as low as the TeV scale.

A. CFB Quark Bilinears

We next consider the bilinears of column 3, which do not arise in MFV. The six quark-bilinears involve one or more factors of Y_e , each of which introduces new off-diagonal structures relative to MFV. One of these operators is harmless since it contributes only to charged currents: O_{6e} might look unsuppressed, but in practice it is subdominant in comparison to the SM tree-level contributions that it competes against.

Operators such as O_{1e} , O_{3e} , O_{4e} and O_{5e} each contain insertions of $Y_e Y_e^\dagger$, for which

$$|Y_e Y_e^\dagger| = \frac{1}{\eta^2} \begin{pmatrix} 3.4 & 14.9 & 11.2 \\ 14.9 & 66.2 & 49.6 \\ 11.2 & 49.6 & 37.1 \end{pmatrix}, \quad (15)$$

where $\eta^2 \simeq 10^6$. They are therefore relatively suppressed.

As an example, consider the $K^0 - \bar{K}^0$ mixing. The operator to be considered is $\frac{1}{\Lambda^2} O_{5e} O_{5e}$, with Λ a parameter characterising the scale of new physics, while operators built from O_{1e} and O_{3e} have additional suppression due to Y_d . The scale Λ is expected to be given by the flavor-breaking scale F of Section II, up to model-dependent factors.

Assuming the same value for the matrix element as the SM operator, and the formalism in [8], we find

$$\frac{\Delta M_K(\text{new})}{\Delta M_K(\text{SM})} \simeq \left(\frac{44 \text{ GeV}}{\Lambda} \right)^2 \cdot \left[\frac{(Y_e Y_e^\dagger)_{21}}{10^{-5}} \right]^2. \quad (16)$$

Because $(Y_e Y_e^\dagger)_{21} \sim 10^{-5}$, this observable receives small corrections, negligible for $\Lambda > 1$ TeV as generically expected already from the MFV operators.

The bilinear O_{2e} contributes for example to $D^0 - \bar{D}^0$ mixing through $O_{2e} O_{2e}$. Each bilinear involves the matrix

$$|Y_d Y_e| = \frac{1}{\eta} \begin{pmatrix} 3 \times 10^{-5} & 2 \times 10^{-8} & 2 \times 10^{-12} \\ 0.003 & 3 \times 10^{-5} & 5 \times 10^{-9} \\ 0.1 & 0.003 & 0.0003 \end{pmatrix} \quad (17)$$

where the 21 entry is very small. Again this process is adequately suppressed with a scale on the order of 1 TeV.

We conclude that the new operators arising in CFB are such that all quark FCNC processes are adequately suppressed by flavor scales as low as 1 TeV — just as in MFV.

B. CFB Charged-Lepton Bilinears

Of the three lepton bilinears of column-3, $O_{1\ell}$ is suppressed by at least two powers of the bottom Yukawa coupling and is harmless. The second operator $O_{2\ell}$, though, because of large off-diagonal entries in $Y_d^\dagger Y_u$, does introduce a stringent constraint on the flavor scale. Diagonalizing Y_e via $Y_e = L^{(e)} \text{diag}(Y_e) R^{(e)\dagger}$ and using the relation in Eq. (3), we have the flavor-changing matrix among the lepton mass eigenstates as

$$\begin{aligned} & \eta |\text{diag}(Y_e) R^{(e)\dagger} \text{diag}(Y_u)^2 R^{(e)}| \\ &= \eta \begin{pmatrix} 2.2 \times 10^{-6} & 2.3 \times 10^{-7} & 2.1 \times 10^{-9} \\ 1.3 \times 10^{-5} & 1.4 \times 10^{-6} & 1.3 \times 10^{-8} \\ 9.9 \times 10^{-6} & 1.0 \times 10^{-6} & 9.7 \times 10^{-9} \end{pmatrix}. \end{aligned} \quad (18)$$

Dressing this operator with a Lorentz structure, we have $1.3 \times 10^{-2} v_W \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} / (\sqrt{2} \Lambda^2)$ or $9.9 \times 10^{-3} v_W \times \bar{\tau}_L \sigma^{\mu\nu} e_R F_{\mu\nu} / (\sqrt{2} \Lambda^2)$ or $1.0 \times 10^{-3} v_W \times \bar{\tau}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu} / (\sqrt{2} \Lambda^2)$. The currently most stringent constraint comes from the rare process $\mu \rightarrow e\gamma$, requiring $\Lambda \gtrsim 10^4$ TeV [9],⁴ far beyond the energy reached by the Large Hadron Collider. The operator $O_{2\ell}$ also leads to a simple relation between the three radiative decays $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ that might become of relevance for future experimental searches [10].

Another constraint arises from the bilinear $O_{3\ell}$. After rotating to the mass eigenstate basis, we have the flavor-changing matrix

$$\begin{aligned} & |R^{(e)\dagger} \text{diag}(Y_u)^2 R^{(e)}| \\ &= \begin{pmatrix} 0.99 & 0.10 & 1.0 \times 10^{-3} \\ 0.10 & 0.01 & 1.0 \times 10^{-4} \\ 1.0 \times 10^{-3} & 1.0 \times 10^{-4} & 1.0 \times 10^{-6} \end{pmatrix}. \end{aligned} \quad (19)$$

The relatively large 12 and 21 entries are again a consequence of large off-diagonal entries in Y_e . They lead to a bound on the flavor scale via the process $\mu^- \rightarrow e^- e^+ e^-$. Employing the dimension-six operator, $0.1 \bar{\mu}_R \gamma_\mu e_R \bar{e} \gamma^\mu e / \Lambda^2$, the constraint is $\Lambda \gtrsim 320$ TeV [9].

Finally we note that the dimension-6 operators formed by composing the bilinears of Table II can also be used at the quantum level, where they can lead to loop corrections to Eq. (3). Composing the bilinear O_{1u} with itself, for example, and employing this operator twice, produces a two-loop correction to the down-quark mass operator $H \bar{Q}_L Y_d d_R$, and therefore to Eq. (3). It is of order $M^4 / (4\pi\Lambda)^4$ times factors of Y_u and Y_u^\dagger , where M is the cutoff (the scale of new physics). As long as $M \lesssim \Lambda$, this will amount to a small correction to Eq. (3).

V. DISCUSSION AND CONCLUSIONS

We have searched for predictive flavor models based on subgroups of $SU(3)^5$, the full flavor group of the quarks and charged leptons in the standard model. Restricting to products of $SU(3)$'s, we were led to a model based on $SU(3)^3$, with its breaking due to two, rather than three, independent 3×3 spurion fields (Yukawa matrices). Thus one of the three Yukawa matrices Y_u , Y_d and Y_e can be expressed in terms of the other two. Taking Y_u and Y_e to be the basic spurion fields, we chose the symmetry assignments of the fermions and the spurions in such a way as to realize the relation $Y_d \propto Y_u Y_e^\dagger$.

This leads to an interesting relation between the charged-lepton mass ratios and quark mass ratios and mixing angles, shown in Eqs. (13) and (14). It also leads to an unusual, strongly off-diagonal structure of the Yukawa matrix Y_e , indicating that large values of the PMNS angles are natural. While we have not yet incorporated neutrino mass into our model, it would require a fine-tuned relation between Y_e and the neutrino mass matrix to avoid the presence of large PMNS angles. We have provided approximate analytic expressions for the charged lepton mass ratios in Eqs. (13) and (14), noting their dependence on quark masses ratios, and commenting on the adjustments that would be required to bring m_μ/m_τ into closer agreement with experiment. The strongly off-diagonal structure of Y_e also plays a role in making the flavor scale Λ much larger than in MFV.

These results emerge from an EFT incorporating the standard model and the higher-dimension operators that correct it. The results follow from the postulated flavor-symmetry structure $Y_d \propto Y_u Y_e^\dagger$. This relation is stable in the presence of SM radiative corrections since these interactions are suppressed by the small couplings of the SM. It is stable even in the framework of the UV completion of the EFT, up to the flavor breaking scale F , providing that these interactions, too, are weakly coupled at this scale. The important underlying issue of the stability of the electroweak scale itself has to be addressed by additional ingredients that we have not discussed here.

Our study is motivated by minimal flavor violation, which also is based on symmetry arguments alone without specifying the UV dynamics involved in flavor generation. Our approach, though, is more restrictive, postulating a smaller broken symmetry in order to obtain predictive relations among the quark and charged-lepton masses and mixing angles. This goal is partly met, with the conclusion that leptonic FCNC processes constrain the general flavor-symmetry-breaking scale to be above 10^4 TeV, well above the energies accessible at the LHC. This appears to confirm a general lesson: more restrictive generalizations of MFV, providing some predictivity without reference to a UV completion, require such large scales. We also found an unexpected result: large mixing angles in the leptonic sector arise naturally, again without explicit reference to the UV completion. In our particular model, FCNC effects are very small in the quark

⁴ Our constraint is fairly conservative in a sense that an operator constructed from the spurions, Y_u and Y_e , will not have this additional factor of η in Eq. (18).

sector but potentially observable in the next generation of charged-lepton flavor-violating experiments, for example the Fermilab Mu2e experiment [10].

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Appendix A: Notation and numerical input

We collect in this Appendix all the relevant expressions that fix the notation and briefly summarise numerical

values for the experimental quantities of relevance to the paper, taken from [11].

We write the dimension-3 mass terms for the charged fermions of the SM in the form $-\bar{\psi}_L^{(i)} M^{(i)} \psi_R^{(i)}$, with $i = e, d, u$. With these conventions, the diagonalization of the mass matrices can be done by introducing three unitary matrices $L^{(i)}$ acting the left-handed fields and three unitary matrices $R^{(i)}$ acting on the right-handed fields. We use the conventions according to which

$$\text{diag } M^{(i)} = L^{(i)\dagger} M^{(i)} R^{(i)}, \quad (\text{A1})$$

which imply that in the mass basis the charged-current weak interactions contain the CKM mixing matrix

$$V_{\text{CKM}} = L^{(u)\dagger} L^{(d)}. \quad (\text{A2})$$

These definitions imply for example that if we choose to write the Lagrangian in a basis in which weak interaction are diagonal (flavour basis), and in which the down-type quarks are diagonal, then the up-type quark flavour eigenstates $|u\rangle$ are related to the mass eigenstates $|\hat{u}\rangle$ as

$$|\hat{u}\rangle = V_{\text{CKM}}|u\rangle. \quad (\text{A3})$$

The conventional way to parameterise a unitary matrix V_{CKM} requires to introduce three angles θ_{ij} and a CP-violating phase δ . In terms of these, we write

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (\text{A4})$$

where $s_{ij} \equiv \sin \theta_{ij}$ and $c_{ij} \equiv \cos \theta_{ij}$. The experimental values are

$$\begin{aligned} \sin \theta_{12} &= 0.2243 \pm 0.0016, \\ \sin \theta_{23} &= 0.0413 \pm 0.0015, \\ \sin \theta_{13} &= 0.0037 \pm 0.0005, \\ \sin 2\beta &= 0.679 \pm 0.019. \end{aligned}$$

The angle $\beta \equiv \arg[-V_{cd}V_{cb}^*/(V_{td}V_{tb}^*)]$ in the CKM unitarity triangle is related to the phase δ .

For the CKM matrix, one could also use the Wolfenstein parametrization,

$$V_{\text{CKM}} = \begin{bmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} \quad (\text{A5})$$

The experimental values are

$$\begin{aligned} \lambda &= 0.22535 \pm 0.00065, \\ A &= 0.811_{-0.012}^{+0.022}, \\ \bar{\rho} \approx \rho &= 0.131_{-0.013}^{+0.026}, \\ \bar{\eta} \approx \eta &= 0.345_{-0.014}^{+0.013}. \end{aligned}$$

Finally, for the masses of the quarks we adopt the convention of considering all of them as defined at the common scale M_Z . We have

$$\begin{aligned} m_t &= 176 \pm 5 \text{ GeV}, \\ m_b &= 2.95 \pm 0.15 \text{ GeV}, \\ m_c &= 0.65 \pm 0.12 \text{ GeV}, \\ m_s &= 0.062 \pm 0.015 \text{ GeV}, \\ m_u &= 0.0017 \pm 0.0005 \text{ GeV}, \\ m_d &= 0.0032 \pm 0.0009 \text{ GeV}. \end{aligned}$$

The masses of the charged leptons are affected by smaller

errors and smaller running effects, both of which we neglect, and are given by

$$\begin{aligned} m_\tau &= 1.78 \text{ GeV}, \\ m_\mu &= 0.106 \text{ GeV}, \\ m_e &= 0.511 \times 10^{-3} \text{ GeV}. \end{aligned}$$

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- [1] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012) [arXiv:1207.7214 [hep-ex]].
 - [2] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012) [arXiv:1207.7235 [hep-ex]].
 - [3] For example, models with $SU(3)$ family symmetries include the following. M. J. Bowick and P. Ramond, Phys. Lett. B **103**, 338 (1981); D. R. T. Jones, G. L. Kane and J. P. Leveille, Nucl. Phys. B **198**, 45 (1982); Z. G. Berezhiani, Phys. Lett. B **150**, 177 (1985); Z. G. Berezhiani and M. Y. Khlopov, Sov. J. Nucl. Phys. **51**, 739 (1990) [Yad. Fiz. **51**, 1157 (1990)]; G. G. Ross, L. Velasco-Sevilla and O. Vives, Nucl. Phys. B **692**, 50 (2004) [hep-ph/0401064]; A. Hernandez Galeana and J. H. Montes de Oca Yemha, Rev. Mex. Fis. **50**, 522 (2004) [hep-ph/0406315]; S. F. King and G. G. Ross, Phys. Lett. B **520**, 243 (2001) [hep-ph/0108112]; S. F. King and G. G. Ross, Phys. Lett. B **574**, 239 (2003) [hep-ph/0307190]; T. Appelquist, Y. Bai and M. Piai, Phys. Lett. B **637**, 245 (2006) [hep-ph/0603104]; T. Appelquist, Y. Bai and M. Piai, Phys. Rev. D **74**, 076001 (2006) [hep-ph/0607174]; B. A. Dobrescu and P. J. Fox, JHEP **0808**, 100 (2008) [arXiv:0805.0822 [hep-ph]]; P. W. Graham and S. Rajendran, Phys. Rev. D **81**, 033002 (2010) [arXiv:0906.4657 [hep-ph]]; Y. Koide and T. Yamashita, Phys. Lett. B **711**, 384 (2012) [arXiv:1203.2028 [hep-ph]]; Y. L. Wu, Phys. Lett. B **714**, 286 (2012) [arXiv:1203.2382 [hep-ph]]; M. C. Chen, M. Fallbacher, M. Ratz and C. Staudt, Phys. Lett. B **718**, 516 (2012) [arXiv:1208.2947 [hep-ph]]; C. S. Fong and E. Nardi, Phys. Rev. D **89**, no. 3, 036008 (2014) [arXiv:1307.4412 [hep-ph]]; M. Baumgart, D. Stolarski and T. Zorawski, Phys. Rev. D **90**, no. 5, 055001 (2014) [arXiv:1403.6118 [hep-ph]]; E. Nardi, arXiv:1503.01476 [hep-ph].
 - [4] For a recent review on the flavor puzzle, see F. Feruglio, arXiv:1503.04071 [hep-ph].
 - [5] R. S. Chivukula and H. Georgi, Phys. Lett. B **188**, 99 (1987).
 - [6] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645**, 155 (2002) [hep-ph/0207036]; K. Agashe, M. Papucci, G. Perez and D. Pirjol, hep-ph/0509117; T. Feldmann and T. Mannel, JHEP **0702**, 067 (2007) [hep-ph/0611095].
 - [7] B. Grinstein, V. Cirigliano, G. Isidori and M. B. Wise, Nucl. Phys. B **763**, 35 (2007) [hep-ph/0608123].
 - [8] A. J. Buras, hep-ph/9806471.
 - [9] A. de Gouvea and P. Vogel, Prog. Part. Nucl. Phys. **71**, 75 (2013) [arXiv:1303.4097 [hep-ph]].
 - [10] R. J. Abrams *et al.* [Mu2e Collaboration], arXiv:1211.7019 [physics.ins-det].
 - [11] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).